FUNDAMENTALS

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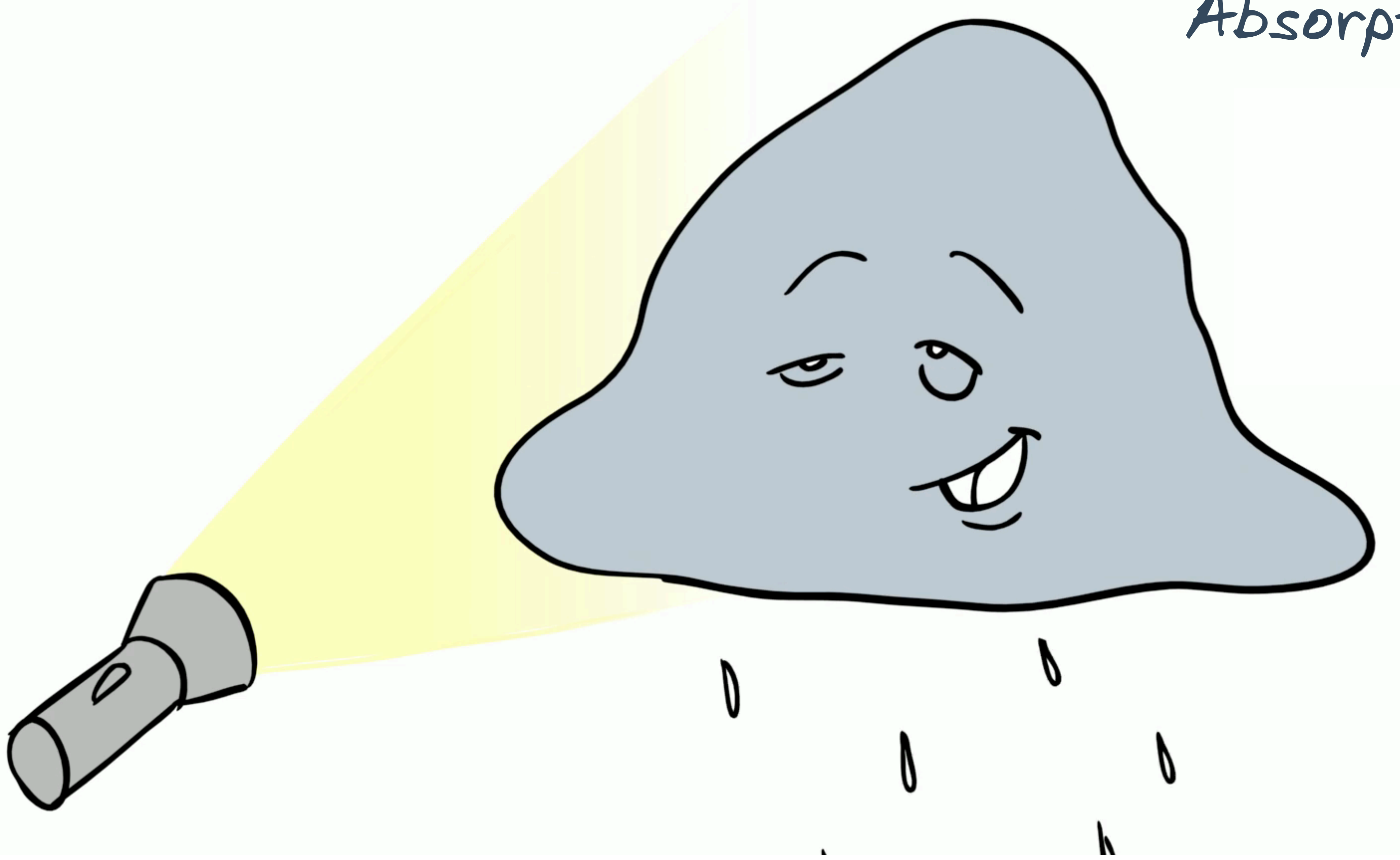
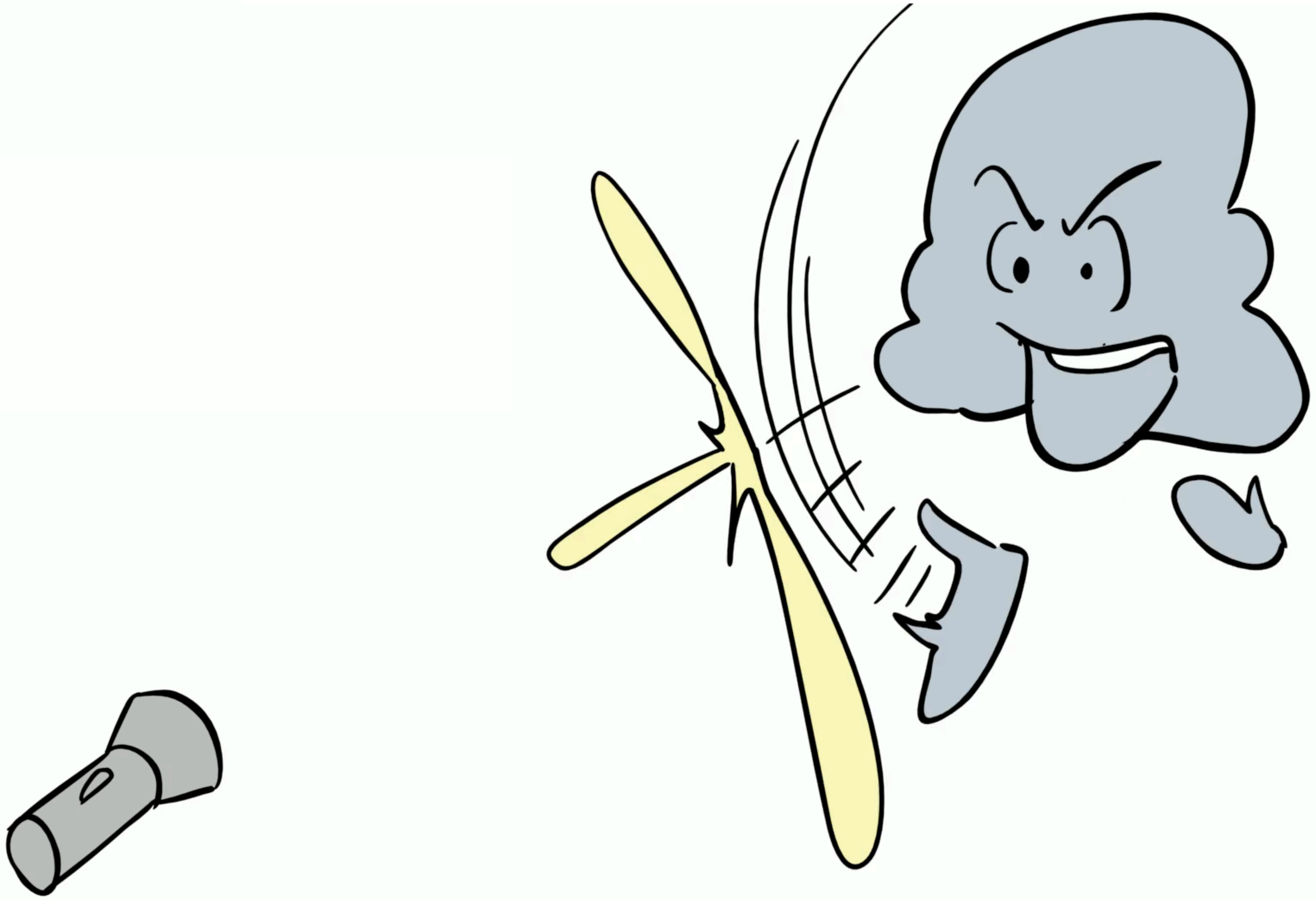
Disney Research

Absorption



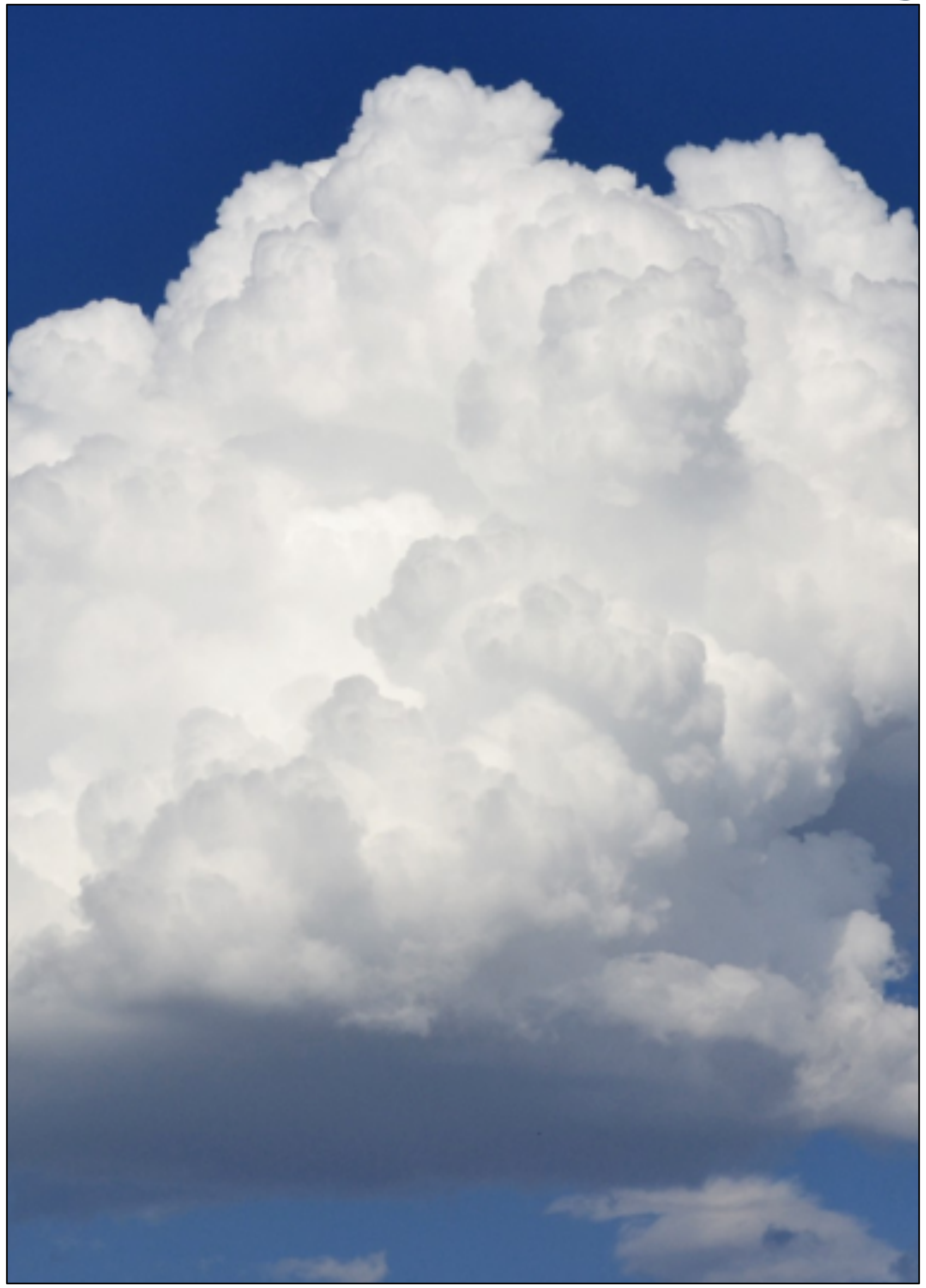
Scattering



Absorption

http://commons.wikimedia.org

Scattering

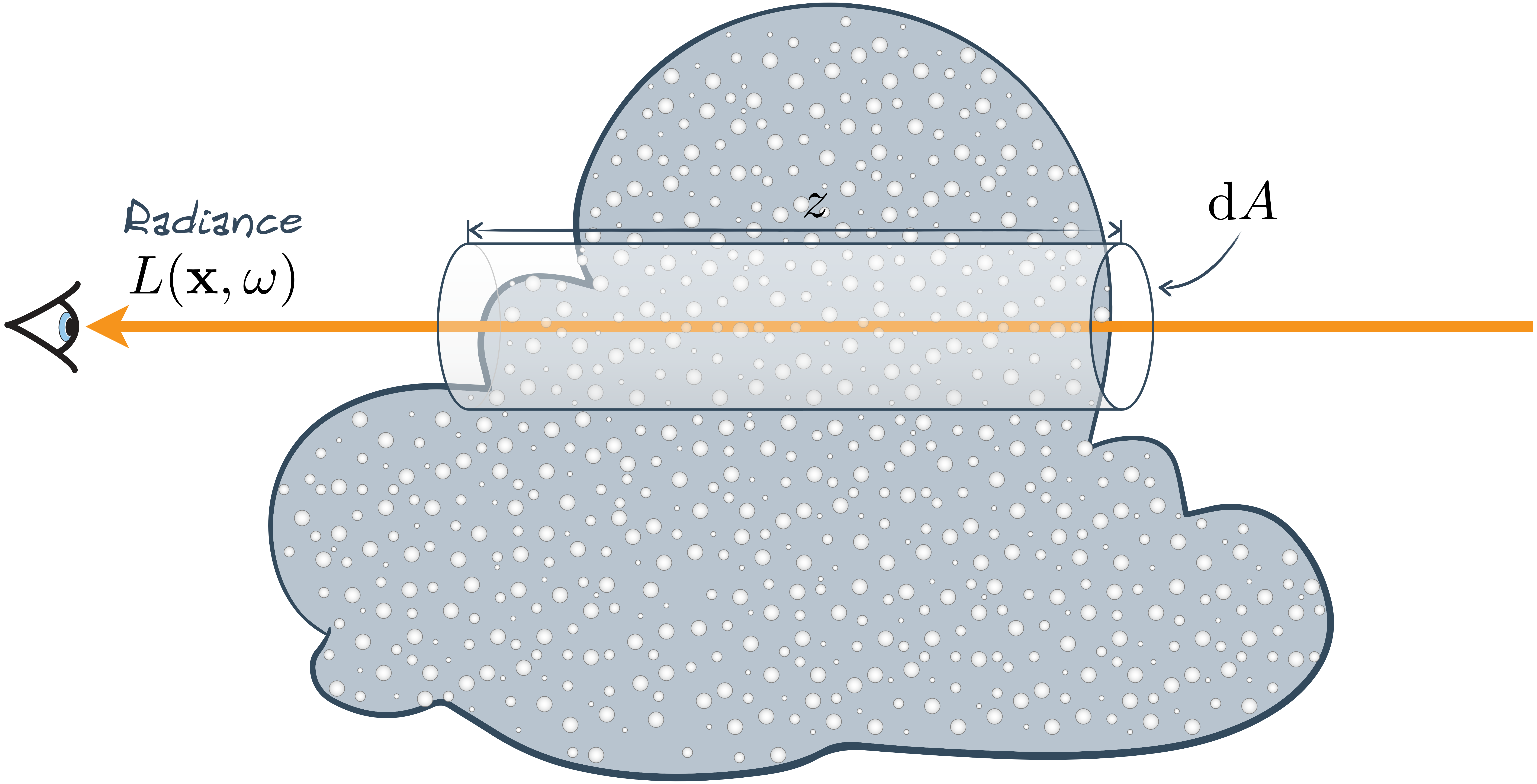


http://coclouds.com

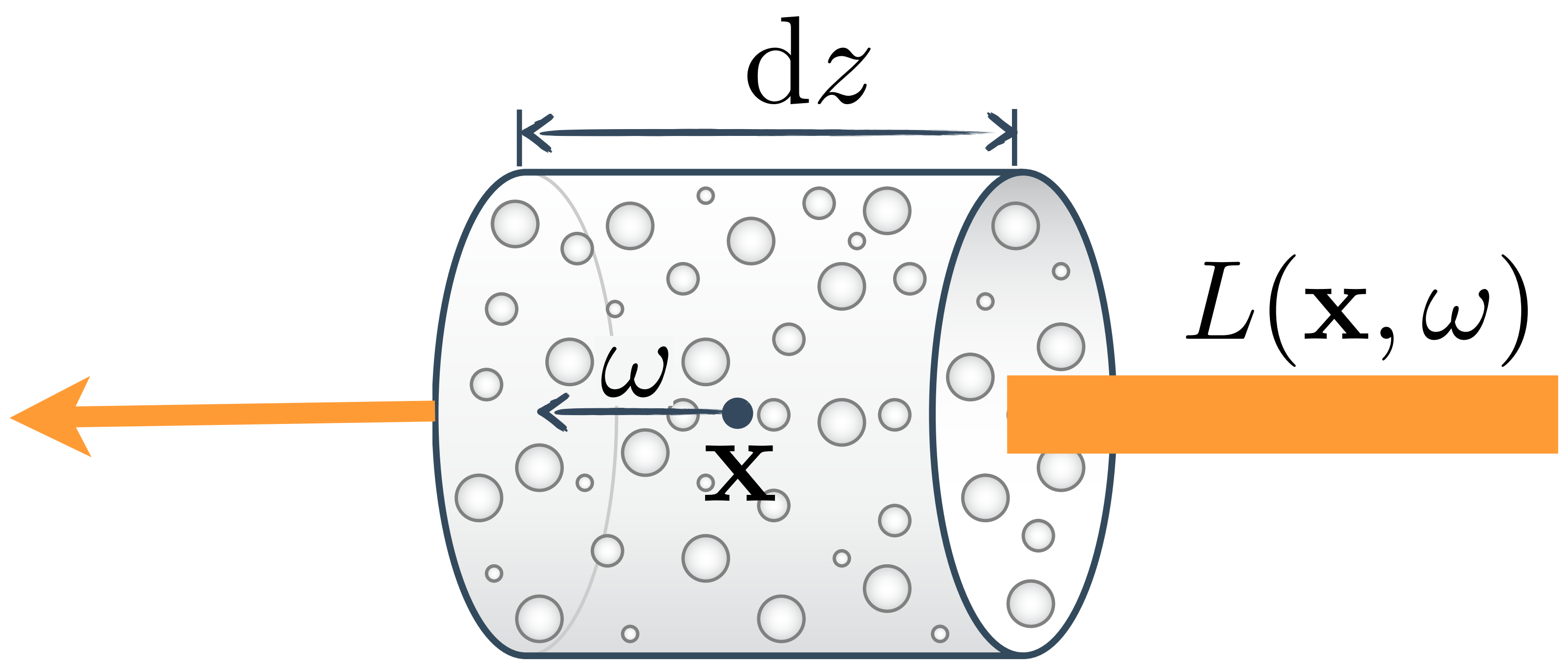
Emission

http://wikipedia.org

RADIATIVE TRANSFER



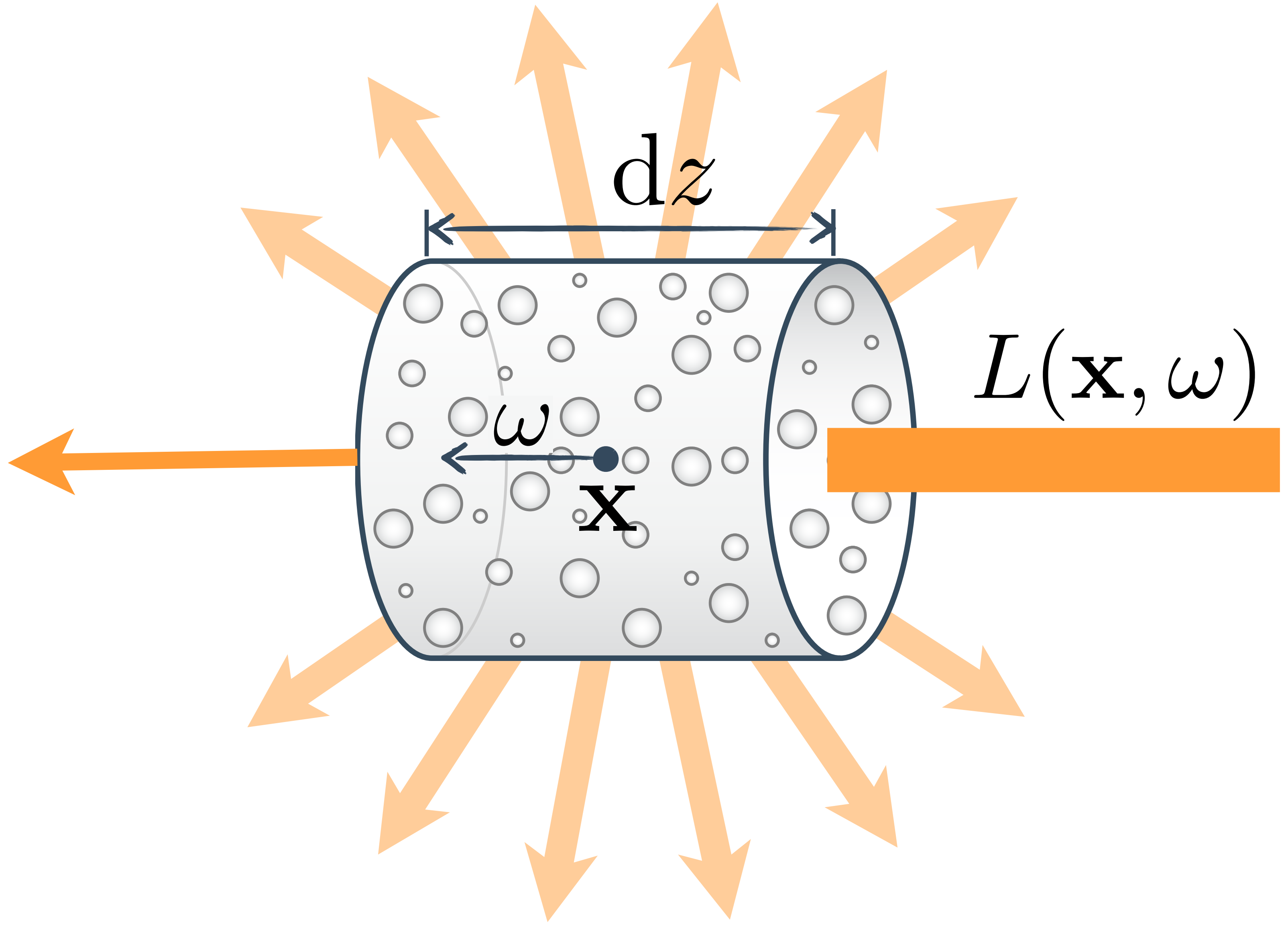
ABSORPTION



$$\frac{\mathrm{d}L}{\mathrm{d}z} = -\mu_\mathrm{a}(\mathbf{x})L(\mathbf{x}, \omega)$$

μ_a – absorption coefficient

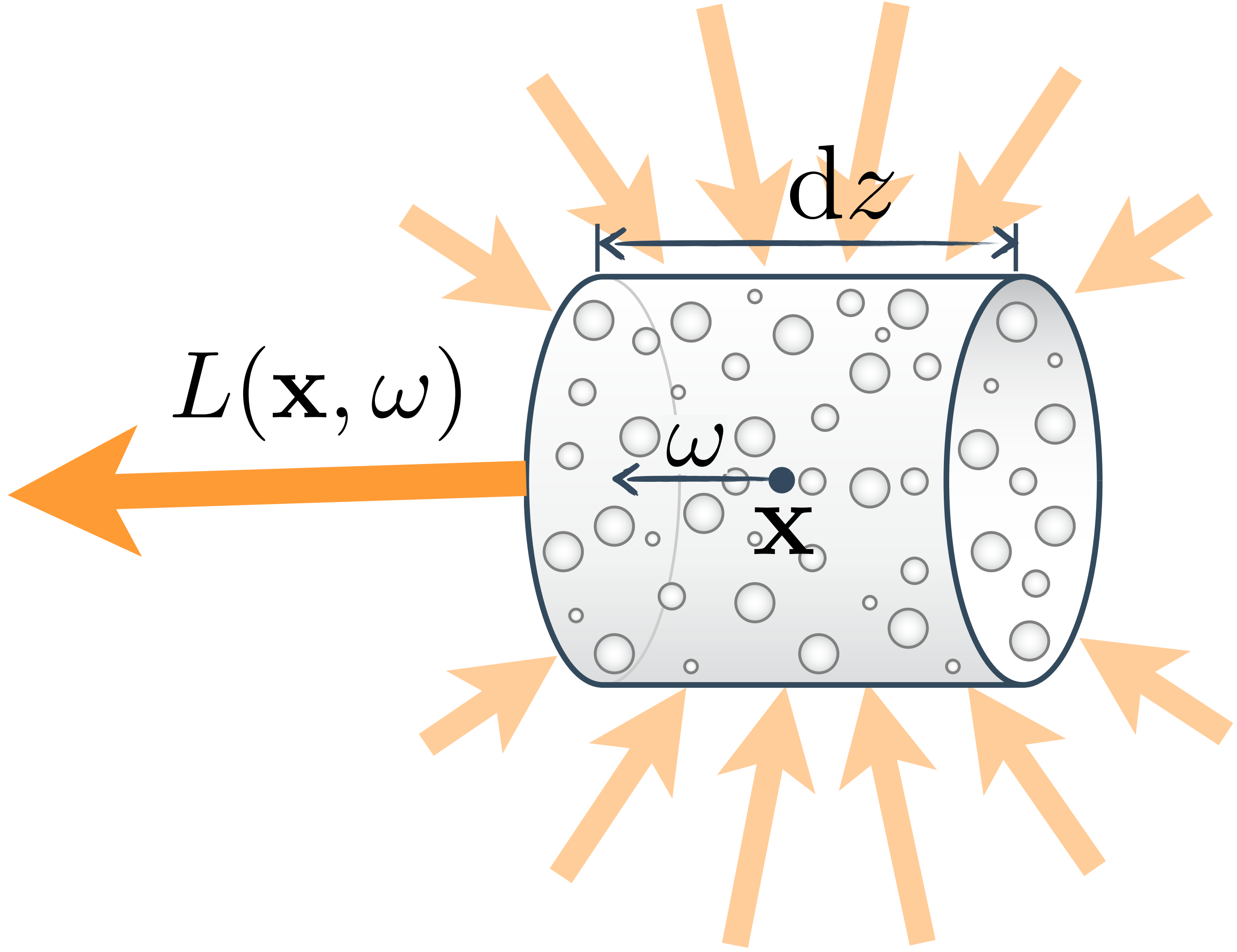
OUT-SCATTERING



$$\frac{\mathrm{d}L}{\mathrm{d}z} = -\mu_{\mathrm{s}}(\mathbf{x})L(\mathbf{x}, \omega)$$

μ_{s} - *scattering coefficient*

IN-SCATTERING



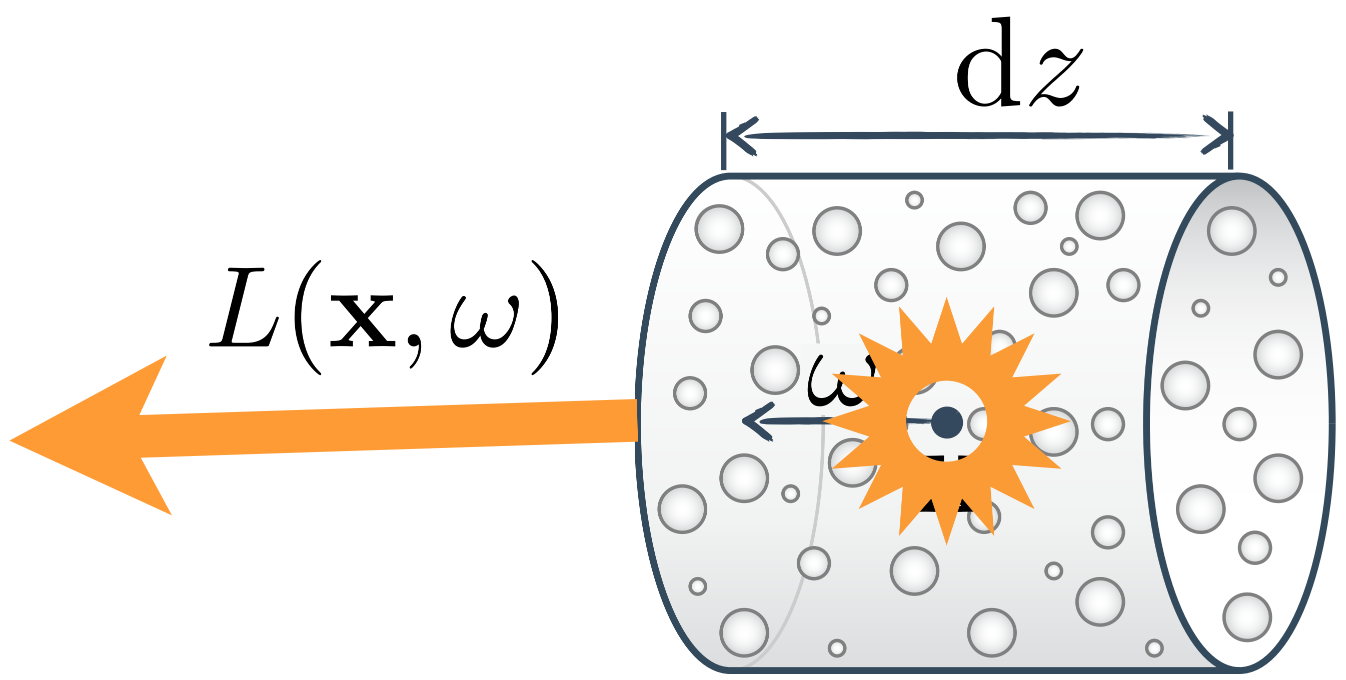
$$\frac{\mathrm{d}L}{\mathrm{d}z} = \mu_\mathrm{s}(\mathbf{x})L_\mathrm{s}(\mathbf{x},\omega)$$

In-scattered radiance

$$L_\mathrm{s}(\mathbf{y},\omega) = \int_{S^2} f_\mathrm{p}(\omega,\bar{\omega})L(\mathbf{y},\bar{\omega})\mathrm{d}\bar{\omega}$$

μ_s – *scattering coefficient*

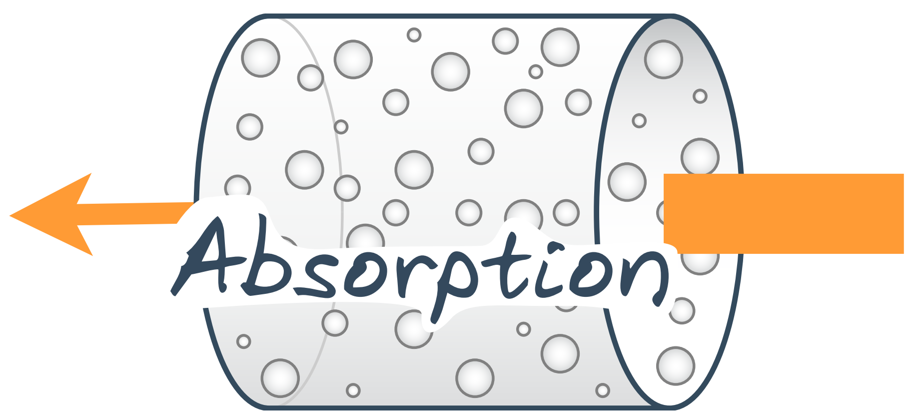
EMISSION



$$\frac{\mathrm{d}L}{\mathrm{d}z} = \mu_\mathrm{a}(\mathbf{x}) L_\mathrm{e}(\mathbf{x}, \omega)$$

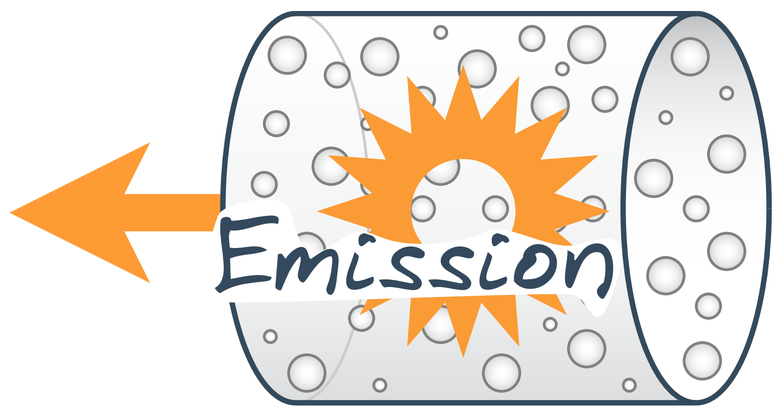
L_e - emitted radiance

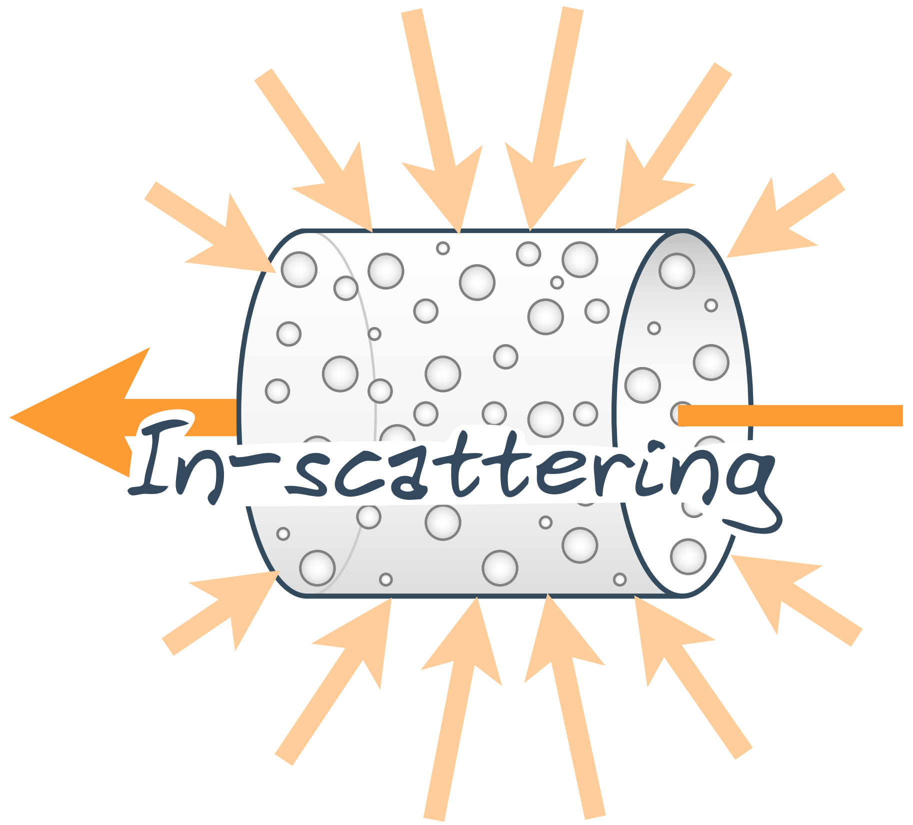
RADIATIVE TRANSFER EQUATION



$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = -\mu_{\mathrm{a}}(\mathbf{x})L(\mathbf{x},\omega) \quad -\mu_{\mathrm{s}}(\mathbf{x})L(\mathbf{x},\omega)$$

$$+\mu_{\mathrm{a}}(\mathbf{x})L_{\mathrm{e}}(\mathbf{x},\omega) + \mu_{\mathrm{s}}(\mathbf{x})L_{\mathrm{s}}(\mathbf{x},\omega)$$





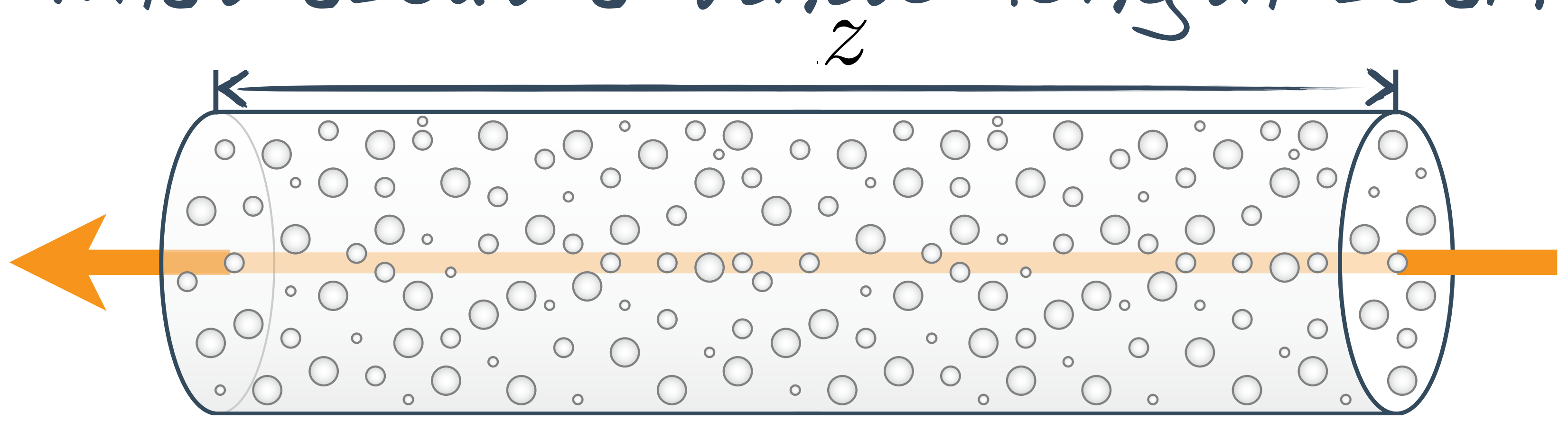
[Chandrasekhar 1960]

RADIATIVE TRANSFER EQUATION

Extinction coefficient $\mu_\mathrm{t}(\mathbf{x}) = \mu_\mathrm{a}(\mathbf{x}) + \mu_\mathrm{s}(\mathbf{x})$

$$\frac{\mathrm{d}L(\mathbf{x},\omega)}{\mathrm{d}z} = \boxed{-\mu_\mathrm{t}(\mathbf{x})L(\mathbf{x},\omega) \quad \text{Losses}}$$
$$\boxed{+\mu_\mathrm{a}(\mathbf{x})L_\mathrm{e}(\mathbf{x},\omega) + \mu_\mathrm{s}(\mathbf{x})L_\mathrm{s}(\mathbf{x},\omega) \quad \text{Gains}}$$

What about a finite-length beam?

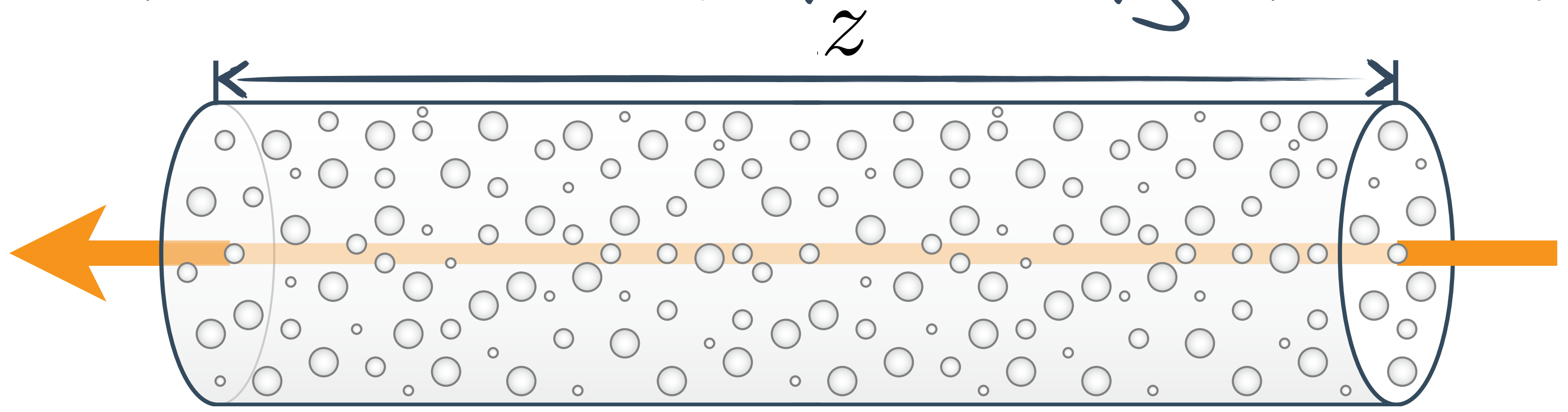


[Chandrasekhar 1960]

RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y}) \Big[\mu_{\mathrm{a}}(\mathbf{y}) L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y}) L_{\mathrm{s}}(\mathbf{y},\omega) \Big] \mathrm{d}y$$

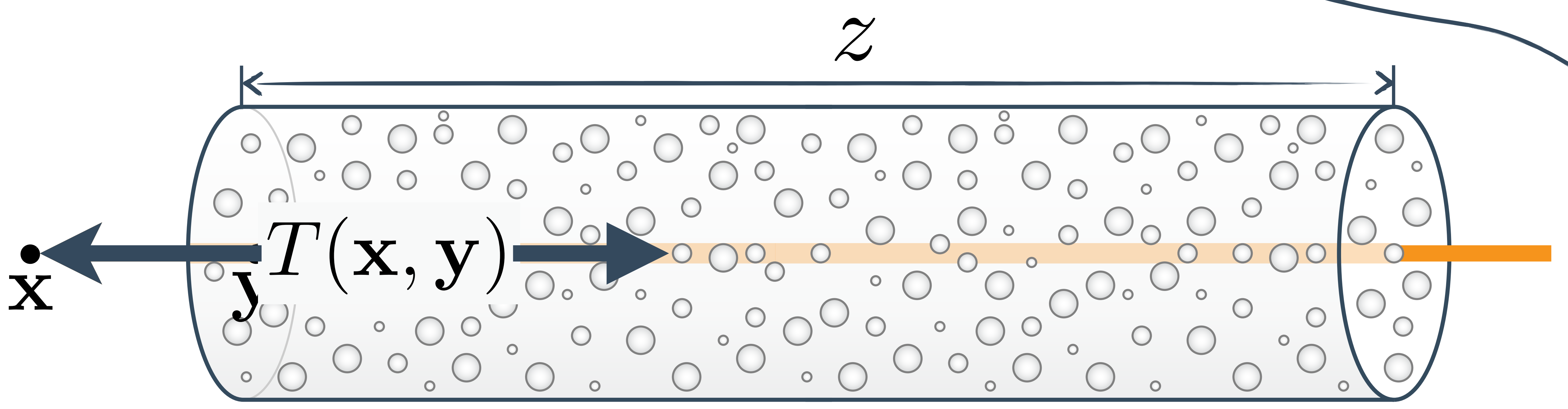
What about a finite-length beam?



RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\Big]\mathrm{d}y$$

Transmittance $T(\mathbf{x},\mathbf{y}) = e^{-\int_0^y \mu_{\mathrm{t}}(\mathbf{s})\mathrm{d}s}$ is the fraction of light that makes it from y to x



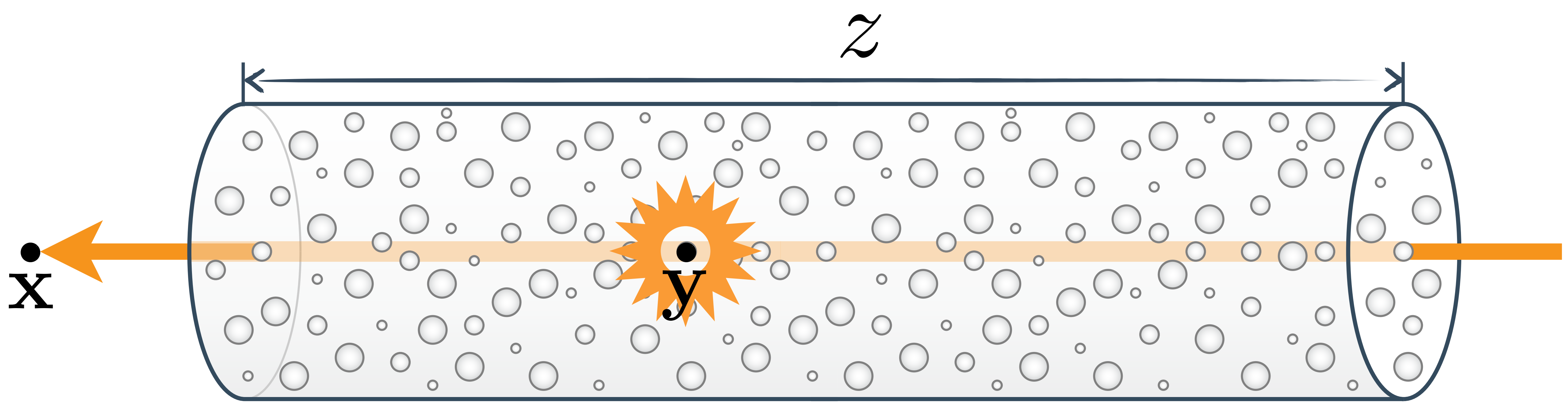
Optical thickness

$$\tau(\mathbf{x},\mathbf{y}) = \int_0^y \mu_{\mathrm{t}}(\mathbf{s})\mathrm{d}s$$

RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y}) \left[\mu_\mathrm{a}(\mathbf{y}) L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y}) L_\mathrm{s}(\mathbf{y},\omega) \right] \mathrm{d}y$$

Emission



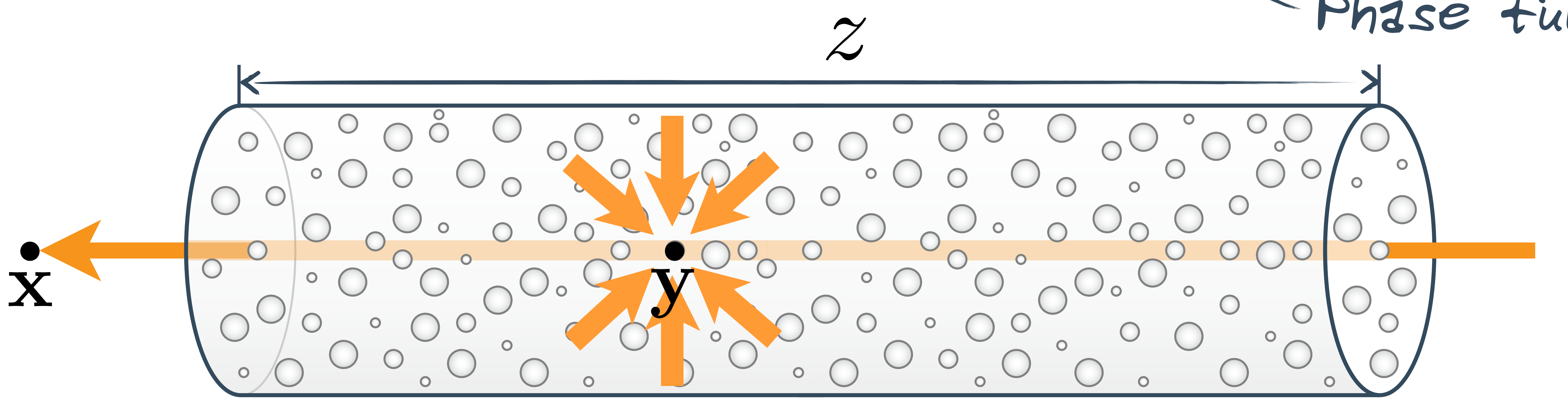
RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\left[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\right]\mathrm{d}y$$

In-scattering

$$L_\mathrm{s}(\mathbf{y},\omega) = \int_{S^2} f_\mathrm{p}(\omega,\bar{\omega})L(\mathbf{y},\bar{\omega})\mathrm{d}\bar{\omega}$$

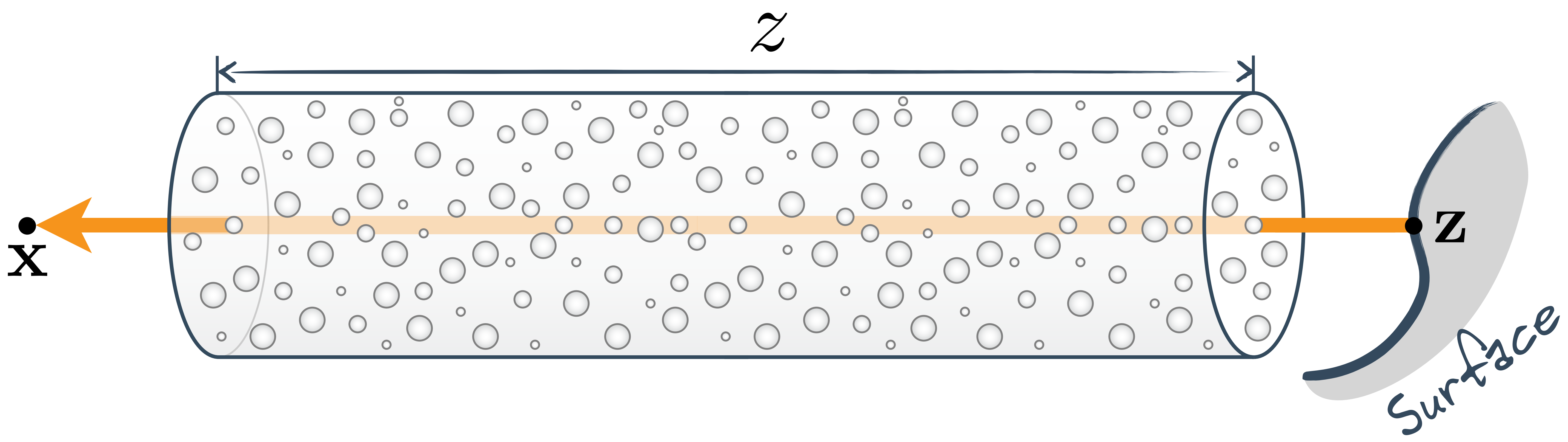
Phase function



RTE — INTEGRAL FORM

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_\mathrm{a}(\mathbf{y})L_\mathrm{e}(\mathbf{y},\omega) + \mu_\mathrm{s}(\mathbf{y})L_\mathrm{s}(\mathbf{y},\omega)\Big]\mathrm{d}y$$
$$+T(\mathbf{x},\mathbf{z})L_\mathrm{o}(\mathbf{z},\omega)$$

Background radiance



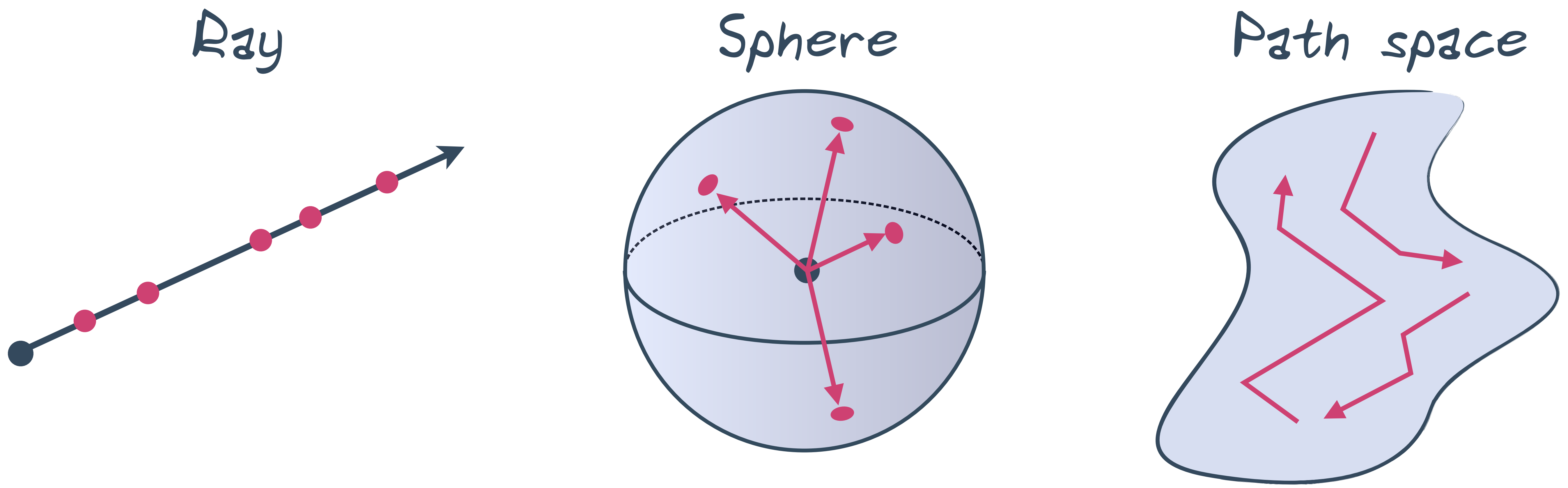
VOLUME RENDERING EQUATION

$$L(\mathbf{x},\omega) = \int_0^z T(\mathbf{x},\mathbf{y})\Big[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\Big]\mathrm{d}y$$
$$+T(\mathbf{x},\mathbf{z})L_{\mathrm{o}}(\mathbf{z},\omega)$$

How do we solve it?

MONTE CARLO INTEGRATION

$$F = \int_{\mathcal{D}} f(x)\,\mathrm{d}x$$



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

Probability density function (PDF)

VRE ESTIMATOR

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T(\mathbf{x},\mathbf{y})}{p(y)} \left[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega) + \mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\right] + \frac{T(\mathbf{x},\mathbf{z})}{P(z)} L_{\mathrm{o}}(\mathbf{z},\omega)$$

$p(y)$ – probability density of distance y

$P(z)$ – probability of exceeding distance z

VRE ESTIMATOR

Transmittance estimation

$$\langle L(\mathbf{x},\omega)\rangle = \frac{T(\mathbf{x},\mathbf{y})}{p(y)}\left[\mu_{\mathrm{a}}(\mathbf{y})L_{\mathrm{e}}(\mathbf{y},\omega)+\mu_{\mathrm{s}}(\mathbf{y})L_{\mathrm{s}}(\mathbf{y},\omega)\right]$$

$$+\frac{T(\mathbf{x},\mathbf{z})}{P(z)}L_{\mathrm{o}}(\mathbf{z},\omega)$$

Distance sampling