PATH CONSTRUCTION

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Solid Angle
Pixel value

\[ I_j = \int_{\mathcal{P}} f_j(\bar{x}) d\bar{x} \]

Pixel estimator

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\bar{x}_i)}{p(\bar{x}_i)} \]
 PATH INTEGRAL FRAMEWORK

Pixel value

\[ I_j = \int_{\mathcal{P}} f_j(\bar{x}) d\bar{x} \]

Pixel estimator

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\bar{x}_i)}{p(\bar{x}_i)} \]

path contribution

path pdf
**Path Integral Framework**

Path contribution

$$f_j(\mathbf{x}) = W_j(x_0, x_1) \left[ \prod_{i} f_s(x_i) G(x_i, x_{i+1}) T(x_i, x_{i+1}) \right] L_e(x_k, x_{k-1})$$

Pixel value

$$I_j = \int_{\mathcal{P}} f_j(\mathbf{x}) d\mathbf{x}$$

Pixel estimator

$$\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\mathbf{x}_i)}{p(\mathbf{x}_i)}$$
**Path Integral Framework**

Path contribution:

\[ f_j(\bar{x}) = W_j(x_0, x_1) \left[ \prod_{i} f_s(x_i) G(x_i, x_{i+1}) T(x_i, x_{i+1}) \right] L_e(x_k, x_{k-1}) \]

Pixel value:

\[ I_j = \int_{\mathcal{P}} f_j(\bar{x}) d\bar{x} \]

Pixel estimator:

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\bar{x}_i)}{p(\bar{x}_i)} \]
Pixel value

\[ I_j = \int_{\mathcal{P}} f_j(\mathbf{x}) \, d\mathbf{x} \]

Pixel estimator

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\mathbf{x}_i)}{p(\mathbf{x}_i)} \]

Path contribution

\[ f_j(\mathbf{x}) = W_j(\mathbf{x}_0, \mathbf{x}_1) \left[ \prod_{i} f_s(\mathbf{x}_i) G(\mathbf{x}_i, \mathbf{x}_{i+1}) T(\mathbf{x}_i, \mathbf{x}_{i+1}) \right] L_e(\mathbf{x}_k, \mathbf{x}_{k-1}) \]
Path contribution

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Pixel value

\[ I_j = \int_{\mathcal{P}} f_j(\mathbf{x}) d\mathbf{x} \]

Pixel estimator

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\mathbf{x}_i)}{p(\mathbf{x}_i)} \]
Path integral framework

Path contribution

\[ f_j(\bar{x}) = W_j(x_0, x_1) \prod_i f_s(x_i)G(x_i, x_{i+1})T(x_i, x_{i+1}) L_e(x_k, x_{k-1}) \]

Pixel value

\[ I_j = \int_P f_j(\bar{x}) d\bar{x} \]

Pixel estimator

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^N \frac{f_j(\bar{x}_i)}{p(\bar{x}_i)} \]
**Path Integral Framework**

#### Path Contribution

Path contribution:

\[
f_j(\vec{x}) = W_j(x_0, x_1) \left[ \prod_{i} f_s(x_i) G(x_i, x_{i+1}) T(x_i, x_{i+1}) \right] L_e(x_{k-1}, x_k)
\]

- **Camera response**
- **BSDF/phase**
- **Geometry**
- **Transmittance**

#### Pixel Value

Pixel value:

\[
I_j = \int_{\mathcal{P}} f_j(\vec{x}) d\vec{x}
\]

#### Pixel Estimator

Pixel estimator:

\[
\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\vec{x}_i)}{p(\vec{x}_i)}
\]
Path contribution

\[
f_j(\vec{x}) = W_j(x_0, x_1) \left[ \prod_i f_s(x_i) G(x_i, x_{i+1}) T(x_i, x_{i+1}) \right] L_e(x_k, x_{k-1})
\]

Pixel value

\[
I_j = \int \rho f_j(\vec{x}) d\vec{x}
\]

Pixel estimator

\[
\langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_j(\vec{x}_i)}{p(\vec{x}_i)}
\]
**PATH INTEGRAL FRAMEWORK**

Pixel value

\[ I_j = \int_P f_j(\bar{x}) d\bar{x} \]

Pixel estimator

\[ \langle I_j \rangle = \frac{1}{N} \sum_{i=1}^{N} f_j(\bar{x}_i) p(\bar{x}_i) \]

Path contribution

\[ f_j(\bar{x}) = W_j(x_0, x_1) \prod_i f_s(x_i) G(x_i, x_{i+1}) T(x_i, x_{i+1}) L_e(x_k, x_{k-1}) \]

- **Camera response**
- **BSDF/phase**
- **Geometry**
- **Transmittance**
- **Emitted radiance**
UNIDIRECTIONAL PATH SAMPLING
UNIDIRECTIONAL PATH SAMPLING

\[ p(t_1|\mathbf{x}_0, \omega_1) \propto T(\mathbf{x}_0, \mathbf{x}_1) \]
UNIDIRECTIONAL PATH SAMPLING

\[ p(\omega_2|x_1) \propto f_s(x_1) \]

\[ \text{direction sampling} \]

\[ x_0 \rightarrow x_1 \rightarrow \omega_2 \]
UNIDIRECTIONAL PATH SAMPLING

\[ p(t_2 | x_1, \omega_2) \propto T(x_1, x_2) \]
UNIDIRECTIONAL PATH SAMPLING

A series of distance and direction sampling decisions
UNIDIRECTIONAL PATH SAMPLING

A series of distance and direction sampling decisions

\[ p(\bar{x}) \propto W_j(x_0, x_1) \left[ \prod_{i} f_s(x_i) G(x_i, x_{i+1}) T(x_i, x_{i+1}) \right] \]

Cannot render illumination from point light sources

High variance when light sources are small

Not importance sampled

Le(x_k, x_{k-1})
EXPLICIT LIGHT SAMPLING

- Path construction

$x_0$, $x_2$
EXPLICIT LIGHT SAMPLING

\[ \begin{align*}
&x_0 \\
&x_1 \\
&x_{2} \\
\end{align*} \]
EXPLICIT: TRANSMITTANCE

\[ p(t_1|x_0) \propto T(x_0, x_1) \]
EXPLICIT: TRANSMITTANCE

\[ T(x_0, x_1) \in [0, 1] \]

\[ G(x_1, x_2) = \frac{1}{||x_1, x_2||^2} \in [0, \infty] \]
EXPLICIT: EQUIANGULAR

\[ p(t_1 | x_0) \propto G(x_1, x_2) = \frac{1}{\|x_1, x_2\|^2} \]
EXPLICIT: EQUIANGULAR

\[ p(t_1 \mid x_0) \propto G(x_1, x_2) = \frac{1}{\|x_1, x_2\|^2} \]

uniform angular distribution
Transmittance sampling, 16 spp

Equiangular sampling, 16 spp
MIS combination

Transmittance sampling

Equiangular sampling

MIS combination
UNIDIRECTIONAL + NEXT EVENT

\[ x_0, x_1, x_2, x_3 \]
UNIDIRECTIONAL + NEXT EVENT
JOINT PATH SAMPLING
JOINT PATH SAMPLING

Joint path sampling:
1) Prescribe joint pdf
2) Derive conditional pdfs via successive joint pdf marginalization
3) Conditionals are obtained in reverse order

TRADITIONAL: prescribes conditional pdfs, no explicit control over joint pdf

JOINT SAMPLING: prescribe joint pdf, conditional pdfs derived from it
Joint Path Sampling

\[ p(x_1, x_2) \propto G(x_0, x_1)G(x_1, x_2)G(x_2, x_3) \]
JOINT PATH SAMPLING

\[ p(t_1) \propto \frac{1}{\| x_3 - x_1 \|} \]
JOINT PATH SAMPLING

 cancels singularity at \( \theta_2 = 0 \)
JOINT PATH SAMPLING

\[ p(t_2) \propto \frac{1}{\|x_3 - x_2\|^2} \]

equiangular pdf
JOINT PATH SAMPLING

\[ p(x_1, x_2) \propto G(x_0, x_1)G(x_1, x_2)G(x_2, x_3) \]
Joint path sampling

\[ p(x_1, x_2) \propto G(x_0, x_1)G(x_1, x_2)G(x_2, x_3)f_s(x_1)f_s(x_2) \]

Joint pdf via tabulation
Equiangular Transmittance
Joint sampling
path lengths 1-8
isotropic phase function

Transmittance  Equiangular  Joint sampling
Joint tabulated path sampling
Transmittance connections
anisotropic phase function
path lengths 1-8
BIDIRECTIONAL PATH TRACING

- Path Construction

\[ x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \]
Sampling technique

\((s, t) = (6, 0)\)
BIDIRECTIONAL PATH TRACING

Sampling technique

\[(s, t) = (5, 1)\]

# vertices from light

# vertices from eye

\((s, t)\) is a sampling technique used in the context of path tracing. The values 5 and 1 likely represent the number of vertices from the light source and the eye, respectively, used in the construction of the path.
Sampling technique

\[(s, t) = (4, 2)\]
Sampling technique

\((s, t) = (3, 3)\)
BIDIRECTIONAL PATH TRACING

Sampling technique

\[(s, t) = (2, 4)\]

# vertices from light
# vertices from eye
Sampling technique

\[(s, t) = (1, 5)\]
BIDIRECTIONAL PATH TRACING

Sampling technique

\( (s, t) = (0, 6) \)
BIDIRECTIONAL PATH TRACING

MONTE CARLO METHODS FOR VOLUMETRIC LIGHT TRANSPORT SIMULATION — PATH CONSTRUCTION
BIDIRECTIONAL PATH TRACING

\[ \begin{align*}
  x_0 & \rightarrow x_1 \\
  x_2 & \rightarrow x_3 \\
  x_4 & \rightarrow x_5
\end{align*} \]
BIDIRECTIONAL PATH TRACING

MONTE CARLO METHODS FOR VOLUMETRIC LIGHT TRANSPORT SIMULATION — PATH CONSTRUCTION
Combined MIS pixel estimator:

\[
\langle I_j \rangle = \sum_{s \geq 0} \sum_{t \geq 0} w_{s,t}(\overline{x}_{i,j}) \frac{f_j(\overline{x}_{i,j})}{p_{s,t}(\overline{x}_{i,j})}
\]
SUMMARY

UNIDIRECTIONAL SAMPLING
- Almost ideal on paper, rarely useful in practice

NEXT EVENT ESTIMATION
- Improvement, but singularity in indirect lighting (reduced convergence rate)

JOINT PATH SAMPLING
- Substantial improvement in the presence of singularities

BIDIRECTIONAL PATH TRACING
- Avoids singularities, more robust thanks to mixing many sampling techniques
- Difficult to implement